*The terms Trust and School (and levels within e.g. governors and trustees) are interchangeable and apply to all schools within the Trust*
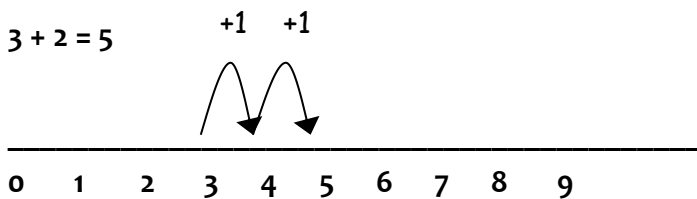
1	ADDITION

STAGE 1 - Recognition that a number represents an amount of objects/things (use images) (From F2 onwards)

Children are encouraged to develop a mental picture of the number system in their heads for use for calculation. They develop ways of recording calculations using pictures, etc.
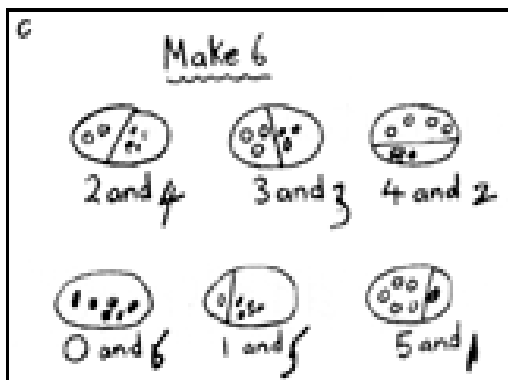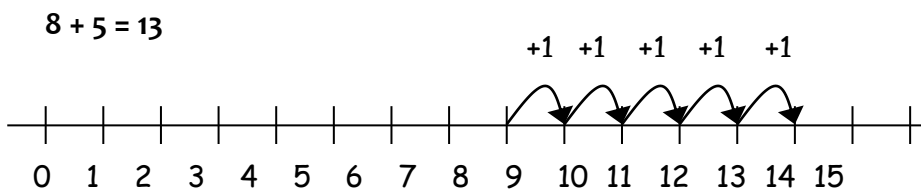

STAGE 2 - Number line to count on

They use number lines and practical resources to support calculation and teachers demonstrate the use of the number line.
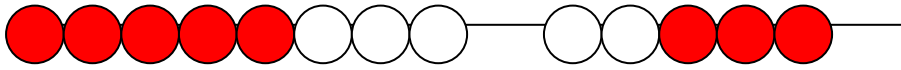
$3 + 2 = 5$

Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$8 + 5 = 13$

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

STAGE 3 - Partitioning (no crossing boundaries) 2-digit numbers (From Year 1 onwards)

Partitioning (least significant digits first in preparation for carrying)

1	5	+	2	2	=	3	7
	5	+		2	=		7
1	0	+	2	0	=	3	0

STAGE 4 - Partitioning (crossing boundaries) 2-digit numbers

Moving onto crossing boundaries e.g.

1	7	+	2	6	=	4	3
	7	+		6	=	1	3
1	0	+	2	0	=	3	0

STAGE 5 - Expanded written method (no crossing boundaries) 2-digit numbers (From Year 2 onwards)

T	U
6	7
+ 2	2
	9
8	0
8	9

STAGE 6 - Expanded written method (crossing boundaries) 2-digit numbers

```
    T  U
    3  7
    5  6
 +  1  3
    8  0
    9  3
```

With regard to money, children from Year 2 should be combining amounts of money to make a particular value and add amounts with the same unit.

STAGE 7 - Formal written method (no crossing boundaries) 2-digit numbers (From Year 3 onwards)

Making the direct link with the expanded written method.

587 + 475

(The expanded method should be used to teach the formal written method of columnar addition - the children shouldn't be expected to work in this way).

```
    T  U
    6  7
 +  2  2
    8  9
```

STAGE 8 - Formal written method (crossing boundaries) 2-digit numbers

```
  T  U
  3  7
  5  6
  9  3
  1
```

With regard to money, children should add money in pounds and pence separately.

Children should them move onto using 3 digits and mixed numbers of digits. Children should still cross boundaries so that they see the connection.

STAGE 9 - Formal written method 3-digit number add 3-digit number (including crossing more than one boundary-just showing children that it's the same regardless of the number of digits)

```
  H T U
  3 9 2
+ 8 5 9
1 2 5 1
  ₁   ₁
```

(Children should use their knowledge of place value to line the numbers up correctly.)

STAGE 10 - Formal written method (mixed number of digits – ensuring children line numbers up properly)

```
  H T U
  4 7 5
+   4 7
  5 2 2
  ₁ ₁
```

(From Year 4)
Extend method to include 4-digit numbers

STAGE 11 - Formal written method including decimals (same number of digits) e.g. 67.3 + 59.2

Year 4 – Using methods, begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds – know that decimal points should line up under each other, particularly when adding mixed amounts such as £3.59 + 78p
Examples:
£4.21 + £3.87
£2.24 + £5.23 + £1.36

```
  T U . t
  3 9 . 4
+ 5 3 . 8
  9 3 . 2
  ₁ ₁
```

STAGE 12 - Formal written method including decimals and mixed number of digits (ensuring children line up digits properly)

H	T	U.	t	h
1	4	3.	8	
+	5	6.	9	4
2	0	0.	7	4
	1	1	1	

With regard to money and measures, children should be adding amounts with decimals.

STAGE 6 - (From Year 5)
Extend to add numbers with more than 4 digits.

Year 5 –Know that decimal points should line up under each other, particularly when adding mixed amounts such as 3.2m + 350cm.
Examples:
£6.72 + £8.56 + £2.30
72.5km + 54.6km
With regard to money and measures, children should be adding amounts with decimals.

Year 6
Children should be answering addition questions which are set in the context of word problems. At this point, children should be confident with all formal methods with varying numbers of decimals and digits.

2	Progression of numbers used in KS1:

FS	Y1	Y2
Say 1 more than a number below 20. Add two 1d numbers using objects. Double 1d numbers using objects.	Numbers to 20 including 0	Fluent in facts such as 3 + 7 = 10 to calculate 30 + 70 = 100. TU + U, TU + tens

3	Progression of numbers used in KS2:

| Y3	Y4	Y5	Y6
TU + TU, developing to HTU + TU or HTU + HTU Extend to 4 digit numbers	HTU + HTU, then ThHTU + ThHTU following on to ThHTU + HTU Decimals to add £ and P and other measurements with decimals	HTU + HTU, then ThHTU + ThHTU Adding numbers with more than 4 digits Decimals to add £ and P and other measurements with decimals	Should be doing everything that Y5 are doing in the context of word problems.

4   SUBTRACTION

STAGE ONE - Recognition that a number represents an amount of objects/things (use images) **(From F2 onwards)**

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures.


STAGE TWO - Number line to show jumps of 1 forward and backwards (children should understand that it works both ways)

They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line to count back.
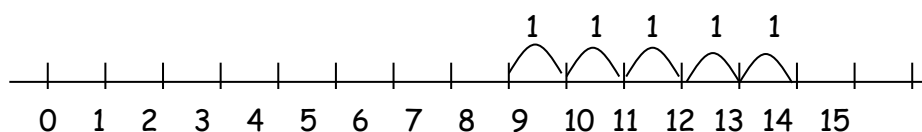
$6 - 3 = 3$
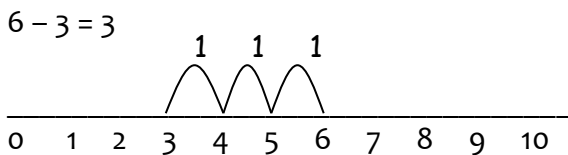
The number line should be used to demonstrate that 6-3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.
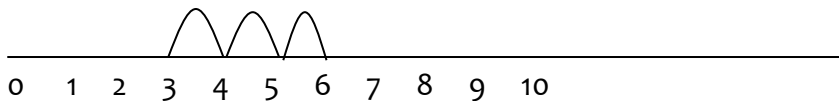
Children then begin to use numbered lines to support their own calculations –using a numbered line to count back in ones.
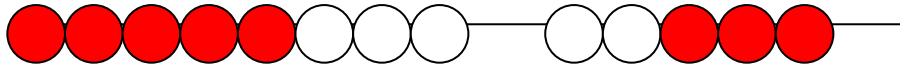
$13 - 5 = 8$

Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.
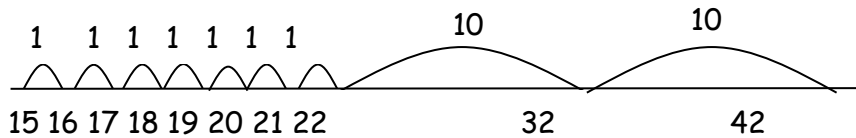
$13 - 5 = 8$

STAGE THREE - Number line to show jumps of 10 and 1 forward and backwards
(From Year 1 onwards)

As the calculations involve using bigger numbers, it is more efficient to count on or back using bigger jumps.

$42 - 15 =$
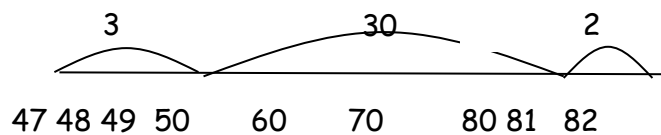

STAGE FOUR - Number line to show jumps of multiples of 10 and 1 forwards and backwards
(From the end of Year 2)

Help children to become more efficient with counting on and back by:
- Adding/subtracting the units in one jump and the tens in one jump

$82 - 47 =$

With regard to money, children should be subtracting amounts of money with the same unit.

STAGE FIVE - Written formal method (no crossing boundaries) 2 digit – 2 digit **(Year 3 onwards)**

Children should be introduced to the columnar subtraction method starting with 2-digit subtract 2-digit.

STAGE SIX - Written formal method (crossing boundaries) 2 digit – 2 digit

Children should be confident with doing this without crossing boundaries before moving onto crossing boundaries.

```
    T   U
   6    1
   7    3
-  3    7
   3    6
```

With regard to money, children should be using £ and p in practical contexts.

STAGE SEVEN - Written formal method (crossing more than one boundary) same no. of digits e.g. 456 – 268

Children should then move onto subtraction with the same number of digits (3) whilst still crossing boundaries.

```
   H    T    U
   2   13    1
   3    4    2
-  1    6    7
   1    7    5
```

STAGE EIGHT - Written formal method (crossing boundaries) varying number of digits (to ensure children line them up properly) e.g. 367 - 98

Before moving onto mixed number of boundaries.

```
   H    T    U
   2   15    1
   3    6    7
-       9    8
   2    6    9
```

STAGE NINE - Written formal method (crossing boundaries) including decimals with the same no. of digits **(Year 4 onwards)**

Children should be using the columnar subtraction method with up to four numbers.

Year 4 – Using methods similar to those above, begin to find the difference between two three-digits amounts of money, with or without adjustment from the pence to the pounds. Know that decimal points should line up under each other. For example:
£9.56 - £3.74
8.5m – 3.7m

```
  T  U . t
 ²3̷ ¹⁵6̷ . 4
-  2  9 . 5
   0  6 . 9
```

STAGE TEN - Written formal method (crossing boundaries) including decimals with mixed no. of digits (not subtracting from nothing)

With regard to money and measures, children should be subtracting amounts of money and measures with decimals.

```
 H  T  U . t  h
 X ¹³4̷ ¹⁴5̷ . ¹3 8
-     9  6 . 4  5
   0  4  8 . 9  3
```

STAGE ELEVEN - Written formal method (crossing boundaries) including decimals with mixed no. of digits (including subtracting from nothing-emphasise taking a 10) **(Year 5 onwards)**

Children should be using the columnar subtraction method with more than 4 digits.

Year 5 – Using the chosen method, find the difference between two decimal fractions with up to three digits and the same number of decimal places. Know that decimal points should line up under each other. For example:
£9.42 - £6.78
72.5km – 4.63km

```
 T  U . t  h
 4 ⁵6̷ . ¹¹7̷ ¹⁰
- 3  3 . 5  6
 1  2 . 6  4
```

Year 6
Children should be answering subtraction questions which are set in the context of word problems. At this point, children should be confident with all formal methods with varying numbers of decimals and digits.

N.B. children should be encouraged to use a number line when subtracting money from a multiple of £1 e.g. £3 - £1.49 or £20 - £12.47

0.01   0.50   1.00

£1.49   £1.50   £2.00   £3.00

0.03   0.50   ·7.00

£12.47   £12.50   £13.00   £20.00
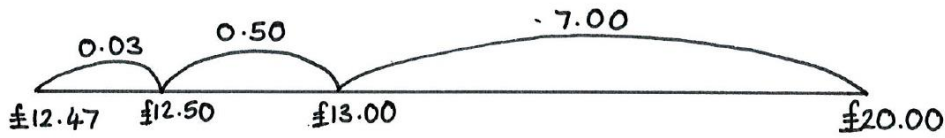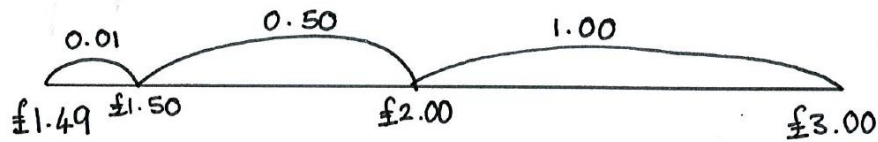
## 5   Progression of numbers used in KS1:

FS	Y1	Y2
Say 1 less than a number below 20. Subtract a 1d number from a number less than 20 using objects. Halve numbers below 20 using objects.	Numbers to 20 including 0	Fluent in facts such as 10 – 7 = 3 to calculate 100-70 = 30. TU - U, TU - tens

6   Progression of numbers used in KS2:

Y3	Y4	Y5	Y6
TU – TU, developing to HTU – TU and HTU – HTU Extend to 4 digit numbers	ThHTU – ThHTU and HTU extend to using various numbers of digits. Use decimals to subtract money and measurements	Subtraction using varying numbers of digits. Use decimals to subtract money and measurements with varying digits after the decimal point.	Should be doing everything that Y5 are doing in the context of word problems.

7   MULTIPLICATION
STAGE ONE Recognition that a number represents an amount of objects/things (use images) **(From F2 onwards)**
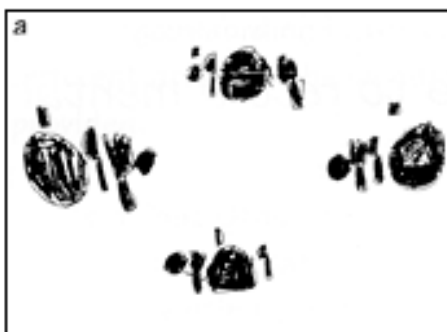
Children will develop their understanding of multiplication and use jottings to support calculation. They will use concrete objects for grouping and arrays with the support of their teacher.

STAGE TWO - Use a number line to show repeated addition (children should be shown the calculation both ways so that they realise that multiplication is commutative) **(From Year 2 onwards)**

Children will become confident at working with arrays and grouping independently and will learn that multiplication is repeated addition.

3 times 5   is   5 + 5 + 5 = 15   or   3 lots of 5   or   5 x 3
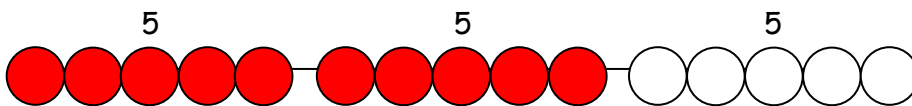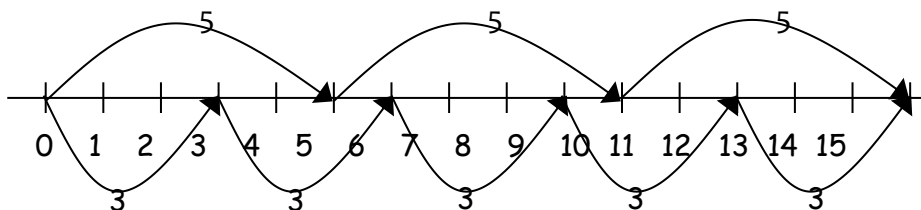
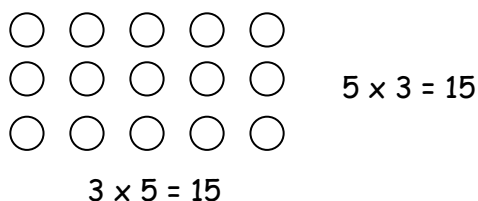Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5
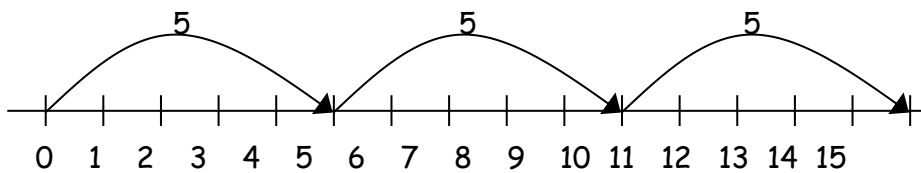
and on a bead bar:

5 x 3 = 5 + 5 + 5

Children will know that 3 x 5 has the same answer as 5 x 3. This can also be shown on a number line.

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

5 x 3 = 15

3 x 5 = 15

STAGE THREE - Informal written method through partitioning 2-digit x 1-digit (2, 3, 4, 5x) **(From Year 2 onwards)**

Children will use informal written methods.

```
2 3 × 4 =

    3 × 4 = 1 2
2 0 × 4 = 8 0
          9 2
```

STAGE FOUR Formal expanded written method 2-digit x 1-digit (5, 3, 4, 8x) **(From Year 3 onwards)**

Children will use informal written methods before moving on to formal written methods for a two digit number multiplied by a one digit number.

```
    T U
    2 6
×     4
    2 4
    8 0
  1 0 4
```

- Always encourage children to approximate first.
  Example, 26 x 4 is approximately 30 x 4 = 120

STAGE FIVE - Formal written method 2-digit x 1-digit (5, 3, 4, 8x)

Leading on to the formal method of calculation (children still to estimate first).

```
    T U
    2 6
×     4
  1 0 4
    2
```

Children to be confident with multiplying by 5, 3, 4 and 8.

STAGE FIVE - Formal written method 2-digit x 1-digit (6, 8, 9, 7x) **(From Year 4 onwards)**

```
    T U
    2 8
x     7
1 9 6
  5
```

Children to be confident with multiplying by 6, 7 and 9 before moving onto multiplying a 1-digit number by a 3-digit number.

STAGE SIX - Formal written method 3-digit x 1-digit and 4-digit x 1-digit

```
  H T U
  3 4 6
x     7
2 1 2 2
  3 4
```

Formal method of multiplication to multiply a four-digit number by a one digit number.

```
Th H T U
 4 2 3 1
x      6
2 5 3 8 6
  1 1
```

STAGE SEVEN - Formal written method 2-digit x 2-digit

Long multiplication for multiplying a two-digit number by a two or three-digit number.

```
  H T U
    2 4
x   1 6
  1 4 4
  2 4 0
  3 8 4
```

```
  H T U
    2 3 5
x     2 7
  1 6 4 5
  4 7 0 0
  6 3 4 5
  1
```

STAGE EIGHT - Formal written method 2-digit x 3-digit and 2-digit x 4-digit **(From Year 6 onwards)**

Th	H	T	U
	4	6	2	5
x		.	3	9
	4	1	6	2	5
1	3	⁵8	²7	⁴5	0
1	'8	0	'3	7	5
	'		'		

STAGE NINE - Formal written method U.t x U and U.th x U

Extend to multiplying numbers with up to two-decimal places by one-digit and two-digit whole numbers.

T	U	.	t
	4	.	9
x	3		
1	4	.	7
	2		

T	U	.	t	h
	4	.	3	6
x	4			
1	7	.	4	4
	1		2	

STAGE TEN - Formal written method U.t x TU and U.th x TU

H	T	U	.	t
		4	.	9
x	3	6		
	2	9	.	4
1	4	³7	.	0
1	²7	6	.	4
	1			

H	T	U	.	t	h
		6	.	3	4
x	5	7			
	4	4	.	3	8
3	1	²7	.	²0	0
3	'6	²1	.	3	8
	1				

14

Progression of numbers used in KS1:

FS	Y1	Y2
	Chdn should make connections between arrays, number patterns and counting in twos, fives and tens.	Multiplication facts for 2, 5 and 10 tables. Count in steps of 2, 3, 5, 10.

Progression of numbers used in KS2

Y3	Y4	Y5	Y6
TU x U	HTU x U	ThHTU x U TU x TU	ThHTU x TU Extend to multiplying numbers with up to two decimal places by a one or two whole digit number

8	**DIVISION**

STAGE ONE - Recognition that a number represents an amount of objects/things (use images to share a selection of objects) (From F2 onwards)

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

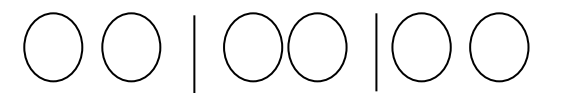### STAGE TWO - Group a selection of objects (From Year 1 onwards)

Children will develop their understanding of division and use jottings to support calculation.
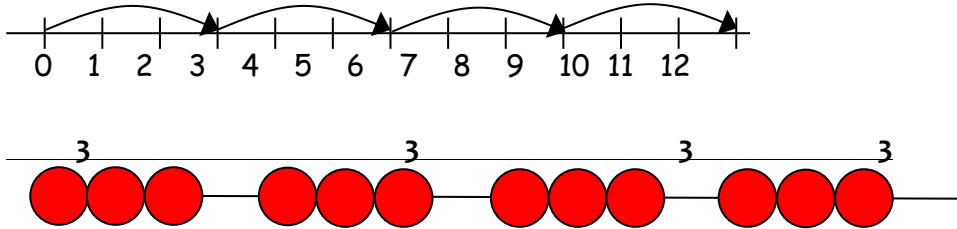
Grouping:
There are 6 sweets, how many people can have 2 sweets each?

Counting on using a number line or bead bar

12 ÷3 = 4

The bead bar will help children with interpreting division calculations such as 10 ÷ 5 as 'how many 5s make 10?'

Use symbols to stand for unknown numbers to complete equations using inverse operations.

□ ÷ 2 = 4   20 ÷ △ = 4   □ ÷ △ = 4

STAGE THREE - Use a number line to count on (repeated addition) (From Year 2 onwards)

Repeated addition using a number line

Children will use an empty number line to support their calculation.

24 ÷ 4 = 6
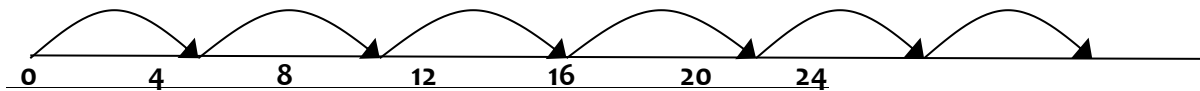
STAGE FOUR - Use a number line to count on-include remainders

Children should also move onto calculations involving remainders.

13 ÷ 4 = 3 r 1

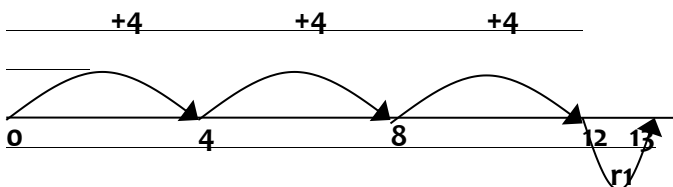Use symbols to stand for unknown numbers to complete equations using inverse operations.

26 ÷ 2 = □   24 ÷ △ = 12   □ ÷ 10 = 8

STAGE FIVE - Informal method (chunking) 2-digit ÷ 1-digit (5, 3, 4, 8 ÷) (From Year 3 onwards)

Children to begin to use informal chunking methods for dividing a two digit number by a one digit number (without remainders) before moving onto formal methods. The chunking method should just be used for a 2-digit number divided by a 1-digit number. Once children are confident the formal method should be taught.

$72 \div 3 = 24$

10	×	3	=	30			6	0
10	×	3	=	30			6	3
1	×	3	=	3			6	6
1	×	3	=	3			6	9
1	×	3	=	3			7	2
1	×	3	=	3				

Without remainders and divided by 5, 3, 4 and 8.

STAGE SIX - Informal method (chunking) 2-digit ÷ 1-digit (5, 3, 4, 8 ÷) with remainders

$79 \div 3 = 26^{r1}$

10	×	3	=	30			6	0
10	×	3	=	30			6	3
1	×	3	=	3			6	6
1	×	3	=	3			6	9
1	×	3	=	3			7	2
1	×	3	=	3			7	5
1	×	3	=	3			7	8
1	×	3	=	3			7	9
				+ 1				

With remainders and divided by 5, 3, 4 and 8.

STAGE SEVEN Informal method (chunking) 2-digit ÷ 1-digit (6, 8, 9, 7 ÷)

$84 \div 6 = 14$

10	×	6	=	60		6	6
1	×	6	=	6		7	2
1	×	6	=	6		7	8
1	×	6	=	6		8	4
1	×	6	=	6			

STAGE EIGHT - Informal method (chunking) 2-digit ÷ 1-digit (6, 8, 9, 7 ÷) with remainders

$$89 \div 7 = 12\ r5$$

$$10 \times 7 = 70 \quad {}^{70}\!>77$$
$$1 \times 7 = 7 \quad {}^{77}\!>84$$
$$1 \times 7 = 7 \quad {}^{84}\!>89$$
$$+ 5$$

STAGE NINE - Formal method 2-digit ÷ 1-digit (From Year 4 onwards)

Children become confident with formal methods for dividing a two-digit or a three-digit number by a one digit number (without remainders).

$$3 \overline{)7\,{}^{1}2} = 24$$

STAGE TEN - Formal method 2-digit ÷ 1-digit with remainders (From Year 5 onwards)

Children divide numbers with up to 4 digits by a one-digit number and interpret the remainders appropriately for the context.

$$3 \overline{)7\,{}^{1}9} = 26\ r1$$

STAGE ELEVEN - Formal method 3-digit ÷ 1-digit and 4-digit ÷ 1 digit with remainders

$$4 \overline{)3\,{}^{3}2\,7} = 081\ r3 \qquad 6 \overline{)5\,{}^{5}2\,{}^{4}4\,{}^{2}9} = 0874\ r5$$

STAGE TWELVE - Formal method 3-digit ÷ 2-digit (From Year 6 onwards)

Children divide numbers with up to 4 digits by a two-digit number using the formal written method of short division and where appropriate interpreting remainders according to the context it is set in.

$$11 \overline{)2\,{}^{2}9\,{}^{7}7} = 027$$

```
          2 8
  1 5 | 4 2 0
        3 0 0
        1 2 0
        1 2 0
```

Children to divide numbers with up to 4 digits by a two digit number and use long division to calculate the answer. Where appropriate, children interpret remainders as whole number remainders, fractions or by rounding, as appropriate to the context.

STAGE THIRTEEN - Formal method 3-digit ÷ 2-digit with remainders

```
          2   8  r12
   1  5 | 4   3   2
          3   0   0
          1   3   2
          1   2   0
              1   2
```

STAGE FOURTEEN - Formal method 3-digit ÷ 2-digit with remainders (showing answer as a fraction)

```
          2   8  12/15  ÷3  4/5
   1  5 | 4   3   2
          3   0   0
          1   3   2
          1   2   0
              1   2
```

STAGE FIFTEEN - Formal method 3-digit ÷ 2-digit with remainders (showing answer as a decimal)

```
          2   8 · 8
   1  5 | 4   3   2
          3   0   0
          1   3   2
          1   2   0
              1   2   0
              1   2   0
```

Pupils begin to divide a number with up to two decimal places by a one or two digit whole number.

Progression of numbers used in KS1:

FS	Y1	Y2
	Chdn should make connections between arrays, number patterns and counting in twos, fives and tens.	Division facts for 2, 5 and 10 tables. Count in steps of 2, 3, 5, 10.

Progression of numbers used in KS2

Y3	Y4	Y5	Y6
TU ÷ U	TU ÷ U and HTU ÷ U	ThHTU ÷ U	ThHTU ÷ TU Divide a number with up to two decimal places by a one or two digit whole number